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TIME: 1½ HRS
M.M.: 50

ANSWER KEY (MAY 2017)

CLASS → XII (N.M.) SUBJECT Maths

(1) $2\cos^{-1}\frac{1}{2} + 3\sin^{-1}\frac{1}{2}$

$= 2\times\frac{\pi}{3} + 3\times\frac{\pi}{6}$

$= \frac{2\pi}{3} + \frac{\pi}{2} = \frac{7\pi}{6}$

(2) Given $B' = -B$

$(ABA')' = (A')'(AB)'$

$= AB'A' = A(-B)A'$

$(ABA')' = -ABA' \Rightarrow ABA'$ is skew symmetric

(3) $\tan^{-1}\left(\frac{2\sin x}{1-\sin^2 x}\right) = \tan^{-1}\left(\frac{2}{\cos x}\right)$

$\Rightarrow \frac{2\sin x}{\cos^2 x} = \frac{2}{\cos x}$

$\Rightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4}$

(4) $\begin{bmatrix}1 & 2 & 1\end{bmatrix}\begin{bmatrix}4\\x\\2x\end{bmatrix} = 0$

$\Rightarrow [4+2x+2x] = [0]$

$\Rightarrow 4+4x=0$

$\Rightarrow 4x=-4 \Rightarrow x=-1$

(5) $A = IA$

$\begin{bmatrix}2 & 1\\1 & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}A$

Applying $R_1 \to R_1 - R_2$, we get

$\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix} = \begin{bmatrix}1 & -1\\0 & 1\end{bmatrix}A.$

Applying $R_2 \to R_2 - R_1$ we get

$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & -1\\-1 & 2\end{bmatrix}A \quad \therefore A^{-1} = \begin{bmatrix}1 & -1\\-1 & 2\end{bmatrix}$

(6) f is Continuous at $x = \frac{\pi}{2}$

$$\lim_{x \to \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \lim_{x \to \frac{\pi}{2}} \frac{k \cos x}{\pi - 2x} = 3$$

$$\Rightarrow \lim_{h \to 0} \frac{k \cos\left(\frac{\pi}{2} - h\right)}{\pi - 2\left(\frac{\pi}{2} - h\right)} = 3 \quad \left[x = \frac{\pi}{2} - h\right]$$

$$\Rightarrow \lim_{h \to 0} \frac{k \sin h}{\pi - \pi + 2h} = 3$$

$$\Rightarrow \lim_{h \to 0} \frac{k \sin h}{2h} = 3$$

$$\Rightarrow \frac{k}{2} \times 1 = 3 \Rightarrow k = 6$$

(7) $6y = x^3 + 2$ — (1)

Differ w.r.to t

$$6 \times \frac{dy}{dt} = 3x^2 \times \frac{dx}{dt} \quad \text{— (2)}$$

Also given that $\frac{dy}{dt} = 8\frac{dx}{dt}$ — (3)

From (2) and (3)

$$6 \times 8 \frac{dx}{dt} = 3x^2 \frac{dx}{dt}$$

$\Rightarrow (x^2 - 16) = 0, \quad \frac{dx}{dt} \neq 0 \quad$ [$\because x$ is not Constant]

$\Rightarrow x = 4, -4$

$6y = 4^3 + 2$ when $x = 4$

$6y = 66 \Rightarrow y = 11$

$6y = (-4)^3 + 2$ when $x = -4$

$6y = -64 + 2$

$\Rightarrow 6y = -62 \Rightarrow y = -\frac{31}{3}$

Reqd pts are $\left(-4, -\frac{31}{3}\right)$ and $(4, 11)$

(8) Let $\cos^{-1}\frac{12}{13} = x$ and $\sin^{-1}\frac{3}{5} = y$

$\Rightarrow \cos x = \frac{12}{13}$ and $\sin y = \frac{3}{5}$

$\sin x = \sqrt{1-\cos^2 x} = \sqrt{1-\frac{144}{169}} = \frac{5}{13}$

$\cos y = \sqrt{1-\sin^2 y} = \sqrt{1-\frac{9}{25}} = \frac{4}{5}$

We know that

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$= \frac{5}{13} \times \frac{4}{5} + \frac{12}{13} \times \frac{3}{5}$

$= \frac{20+36}{65} = \frac{56}{65}$

$x+y = \sin^{-1}\frac{56}{65} \Rightarrow \cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{56}{65}$

(9) Let $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2}$ is the required matrix

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

On equating corresponding elements we get

$a+4b=-7 \quad -(1) \qquad c+4d=2 \quad -(4)$

$2a+5b=-8 \quad -(2) \qquad 2c+5d=4 \quad -(5)$

$3a+6b=-9 \quad -(3) \qquad 3c+6d=6 \quad -(6)$

$(2) - 2\times(1)$ gives $5b-8b = -8+14$

$\Rightarrow -3b = 6 \Rightarrow b=-2$

From (1) we get $a-8=-7 \Rightarrow a=1$

Also $(5) - 2\times(4)$ gives $5d-8d = 4-4$

$\Rightarrow -3d=0 \Rightarrow d=0$

From (4) we get $c+0=2 \Rightarrow c=2$

$\therefore X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

(10) Let

$\Delta = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

Operating $R_1 \to R_1+R_2+R_3$ we get

$\Delta = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

Taking $a+b+c$ common from R_1

$$\Delta = (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Operating $C_1 \to C_1 - C_3$ we get

$$\Delta = (a+b+c)\begin{vmatrix} 0 & 1 & 1 \\ 0 & b-c-a & 2b \\ c+a+b & 2c & c-a-b \end{vmatrix}$$

Expanding along C_1

$$\Delta = (a+b+c)(a+b+c)\begin{vmatrix} 1 & 1 \\ b-c-a & 2b \end{vmatrix}$$

$$\Delta = (a+b+c)^2[2b-(b-c-a)]$$

$$= (a+b+c)^2(b+c+a) = (a+b+c)^3$$

(11) $A^2 = A \cdot A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

L.H.S. $= A^2 - 5A + 7I$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

(12) $y = \log(\log x)$ Differ. w.r.t. x

$$y_1 = \frac{1}{\log x} \times \frac{1}{x} = \frac{1}{x \log x}$$

Again differ w.r.t. x

$$y_2 = -\frac{1}{(x\log x)^2} \frac{d}{dx}[x \log x]$$

$$= -\frac{1}{(x\log x)^2}\left[x \times \frac{1}{x} + \log x \times 1\right]$$

$$= -\frac{1+\log x}{(x \log x)^2}$$

(13) $y = x^{\sin x} + (\sin x)^{\cos x}$

$y = A + B$ where $A = x^{\sin x}$, $B = (\sin x)^{\cos x}$

$$\Rightarrow \frac{dy}{dx} = \frac{dA}{dx} + \frac{dB}{dx}$$

$A = x^{\sin x}$ $\qquad$ $B = (\sin x)^{\cos x}$

$\log A = \sin x \log x$ $\qquad$ $\Rightarrow \log B = \cos x \log(\sin x)$

Differ w.r.t. to x

$\frac{1}{A} \times \frac{dA}{dx} = \sin x \times \frac{1}{x} + \log x (\cos x)$ $\qquad$ $\frac{1}{B} \times \frac{dB}{dx} = \cos x \times \frac{1}{\sin x}(\cos x) + \log \sin x(-\sin x)$

$\Rightarrow \frac{dA}{dx} = A\left[\frac{\sin x}{x} + \cos x . \log x\right]$ $\qquad$ $\Rightarrow \frac{dB}{dx} = B\left[\cos x \cot x - \sin x \log \sin x\right]$

$\Rightarrow \frac{dA}{dx} = x^{\sin x}\left[\frac{\sin x + x \cos x \log x}{x}\right]$ $\qquad$ $\Rightarrow \frac{dB}{dx} = (\sin x)^{\cos x}\left[\cos x \cot x - \sin x \log \sin x\right]$

$\therefore \frac{dy}{dx} = x^{\sin x}\left[\frac{\sin x + x \cos x \log x}{x}\right] + (\sin x)^{\cos x}\left[\cos x \cot x - \sin x \log \sin x\right]$

(14) Given equations can be expressed as $AX = B$

where $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix} = 1(1+3) + 1(2+3) + 1(2-1)$

$= 4+5+1 = 10 \neq 0$ $\quad A^{-1}$ exists

$A_{11} = 4$, $A_{12} = -5$ $\quad A_{13} = 1$, $A_{21} = 2$, $A_{22} = 0$, $A_{23} = -2$

$A_{31} = 2$, $A_{32} = 5$ $\quad A_{33} = 3$

$\text{adj } A = \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}' = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$

$AX = B \Rightarrow X = A^{-1}B$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 16+0+4 \\ -20+0+10 \\ 4+0+6 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

$\Rightarrow x = 2, \; y = -1, \; z = 1$

(15) $f(x) = (x^2 - 2x)^2 \Rightarrow f'(x) = 2(x^2 - 2x)(2x-2) = 4x(x-2)(x-1)$

$f'(x) = 0 \Rightarrow x = 0, 1, 2$ $\quad$ Intervals are $(-\infty, 0)$ $(0,1)$ $(1,2)$, $(2,\infty)$

When $x \in (-\infty, 0)$ $\quad f'(x) = 4(-)(-)(-) < 0$

When $x \in (0,1)$, $\quad f'(x) = 4(+)(-)(-) > 0$

When $x \in (1,2)$, $\quad f'(x) = 4(+)(-)(+) < 0$

When $x \in (2,\infty)$ $\quad f'(x) = 4(+)(+)(+) > 0$

$\therefore f(x)$ is increasing function for $x \in (0,1) \cup (2,\infty)$