SUMMER VACATION ASSIGNMENTS (MATHS)

CLASS-XII

1) If $y=\cot^{-1}\left(\sqrt{\cos x}\right)-\tan^{-1}\left(\sqrt{\cos x}\right)$, prove that $\sin y = \tan^2 \frac{x}{2}$

2) Prove that $\cos^{-1}\left(\frac{\cos\ \alpha + \cos \beta}{1+\cos\ \alpha \cos \beta}\right) = 2\tan^{-1}\left(\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\right).$

3) Prove the following:

a) $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$

b) $\frac{1}{2}\tan^{-1}x = \cos^{-1}\left(\sqrt{\frac{1+\sqrt{1+x^2}}{2\sqrt{1+x^2}}}\right).$

4) Show that:

a) $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$

b) $2\tan^{-1}(-3) = -\frac{\pi}{2}+\tan^{-1}\left(-\frac{4}{3}\right)$

c) $\sin\left(2\tan^{-1}\frac{1}{3}\right) + \cos\left(\tan^{-1}2\sqrt{2}\right) = \frac{14}{15}$

d) $\cos\left(2\tan^{-1}\frac{1}{7}\right) = \sin\left(4\tan^{-1}\frac{1}{3}\right)$

5) Solve :

a) $\cos^{-1}(\sin(\cos^{-1} x)) = \frac{\pi}{3}$

b) $3\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$

6) a) Evaluate $\tan\left(2\tan^{-1}\frac{1}{5}-\frac{\pi}{4}\right).$

b) Prove that $\cot\left(\frac{\pi}{4}-2\cot^{-1}3\right) = 7.$

c) Prove that $)2\sin^{-1}\frac{3}{5}-\tan^{-1}\frac{17}{31} = \frac{\pi}{4}.$

7) Prove the following:

$$a)\ \cos^{-1}\left(\frac{12}{13}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\sin^{-1}\left(\frac{56}{65}\right)$$

$$b)\ \sin^{-1}\left(\frac{4}{5}\right)+\sin^{-1}\left(\frac{5}{13}\right)+\sin^{-1}\left(\frac{16}{65}\right)=\frac{\pi}{2}$$

8) Prove that : $4\left(\cot^{-1}3+\operatorname{cosec}^{-1}\sqrt{5}\right)=\pi.$

9) Solve the following equations for x:

$$1)\ \sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$$

$$2)\ \sin^{-1}x+\sin^{-1}(1-x)=\cos^{-1}x$$

$$3)\ \sin^{-1}\left(\frac{5}{x}\right)+\sin^{-1}\left(\frac{12}{x}\right)=\frac{\pi}{2}.$$

10) If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$, prove that $x^2+y^2+z^2+2xyz=1$.

11) If $a_1, a_2, a_{3......} a_n$ is an arithmetic progression with common difference d, then find the value of

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+\tan^{-1}\left(\frac{d}{1+a_3a_4}\right)+....+\tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right].$$

12) Prove

$$\tan^{-1}\left(\frac{1}{\sqrt{3}}\tan\frac{x}{2}\right)=\frac{1}{2}\cos^{-1}\left(\frac{1+2\cos x}{2+\cos x}\right)$$

13) Prove the following:

$$a)\ \sin^{-1}\frac{1}{\sqrt{5}}+\sin^{-1}\frac{2}{\sqrt{5}}=\frac{\pi}{2}$$

$$b)\ \cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}=\cos^{-1}\frac{33}{65}$$

$$c)\ \cos^{-1}\frac{3}{5}+\sin^{-1}\frac{12}{13}=\sin^{-1}\frac{56}{65}$$

$$d)\ \tan^{-1}\frac{1}{3}+\sec^{-1}\frac{\sqrt{5}}{2}=\frac{\pi}{4}$$

$$e)\ \cos^{-1}\frac{4}{5}+\tan^{-1}\frac{3}{5}=\tan^{-1}\frac{27}{11}$$

$$f)\ \cos^{-1}\frac{5}{\sqrt{41}}+\cot^{-1}\frac{4}{5}=\frac{\pi}{2}$$

$$g)\ \sin^{-1}\frac{1}{\sqrt{17}}+\cos^{-1}\frac{9}{\sqrt{85}}=\tan^{-1}\frac{1}{2}.$$

14) Solve the following equations for X.

a) $\tan^{-1}(2+x)+\tan^{-1}(2-x)=\tan^{-1}\frac{2}{3}$

b) $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}$

c) $\cos(\sin^{-1}x)=\frac{1}{9}$

d) $\sin^{-1}(6x)+\sin^{-1}(6\sqrt{3}x)=-\frac{\pi}{2}$

15) If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi$, prove that $x+y+z=xyz$.

16) If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\frac{\pi}{2}$, prove that $xy+yz+zx=1$.

17) If the angle C of a triangle is right angle, prove that $\tan^{-1}\frac{a}{b+c}+\tan^{-1}\frac{b}{c+a}=\frac{\pi}{4}$, where a,b,c are the lengths of the sides of the triangle opposite to the angles A,B,C respectively.

18) Solve the following equations for x:

a) $\tan^{-1}x=\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

b) $\cos(\sin^{-1}x)=\frac{1}{2}$

c) $4\sin^{-1}x+\cos^{-1}x=\pi$

d) $\sin^{-1}x-\cos^{-1}x=\frac{\pi}{6}$

e) $3\tan^{-1}x+\cot^{-1}x=\pi.$

19) a) If $\tan^{-1}x=\frac{\pi}{10}$ for some $x\in$ **R** find the value of $\cot^{-1}x.$

b) If $\sin^{-1}x+\sin^{-1}y=\frac{\pi}{2}$, then find the value of $\cos^{-1}x+\cos^{-1}y.$

20) If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\pi$, then prove that

$$x^4+y^4+z^4+4x^2y^2z^2=2(x^2y^2+y^2z^2+z^2x^2)$$

21) Solve the following equations for x:

a) $\tan^{-1}\frac{x+1}{x-1}+\tan^{-1}\frac{x-1}{x}=\tan^{-1}(-7)$

b) $\tan^{-1}\frac{1}{2x+1}+\tan^{-1}\frac{1}{4x+1}=\tan^{-1}\frac{2}{x^2}$

22) Solve the following simultaneous equations:

$$\sin^{-1}x+\sin^{-1}y=\frac{2\pi}{3},\cos^{-1}x-\cos^{-1}y=\frac{\pi}{3}$$

23)

a) $\tan(\cos^{-1} x) = \sin\left(\cot^{-1}\dfrac{1}{2}\right)$

b) $\sin(\cot^{-1}(1+x)) = \cos(\tan^{-1} x)$

24) Evaluate :

$$\cos^{-1} x + \cos^{-1}\left(\frac{x}{2} + \frac{1}{2}\sqrt{3-3x^2}\right), x \in \left[\frac{1}{2}, 1\right]$$

25) Prove the following:

$$\sin^{-1}\frac{x}{\sqrt{1+x^2}} + \cos^{-1}\frac{x+1}{\sqrt{x^2+2x+2}} = \tan^{-1}(x^2+x+1).$$

26) Find the value of $\cot^{-1} 9 + \operatorname{cosec}^{-1}\dfrac{\sqrt{41}}{4}$.

27) If $\cos^{-1}\dfrac{x}{2} + \cos^{-1}\dfrac{y}{3} = \alpha$, prove that $9x^2 - 12xy\cos\alpha + 4y^2 = 36\sin^2\alpha$.

28) Find the matrix A such that $\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix} A = \begin{pmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{pmatrix}$

29) a) Let A and B be square matrices of same order . Does $(A+B)^2 = A^2+2AB+B^2$ hold? If not, why?

b) Let A and B be square matrices of order 3x3. Is $(AB)^2 = A^2B^2$? Give reason.

30)Using properties of determinants , solve the following equations for x.

1) $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$ 2) $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$

31) Prove that $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$

32) Prove that:

$$\begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix} = 2+4\sin 2x.$$

33) In a triangle ABC if $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A+\sin^2 A & \sin B+\sin^2 B & \sin C+\sin^2 C \end{vmatrix} = 0$, then prove that Δ ABC is an isosceles triangle.

34) If p,q,r are not in G.P. and

$$\begin{vmatrix} 1 & \frac{q}{p} & \alpha+\frac{q}{p} \\ 1 & \frac{r}{q} & \alpha+\frac{r}{q} \\ p\alpha+q & q\alpha+r & 0 \end{vmatrix} = o,$$ show that $p\alpha^2 + 2q\alpha + r = 0.$

35) The management committee of a residential colony decided to award some of its members (say x) for honesty , some (say y) for helping others (say z) for supervising the workers to keep the colony neat and clean . The sum of all the awardees is 12. Three times the sum of awardees for helping others and supervision added to two times the number of awardees for honesty is 33. If the sum of number of awardees for honesty and supervision is the twice the number of awardees for helping others , using matrix method , find the number of awardees of each category . Apart from these values namely, honesty, helping others and supervision , suggest one more value which the management of the colony must include for awards.

36) If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$,find A^{-1}.Using A^{-1},solve the system of linear equations:

$x-2y=10, 2x-y-z=8, -2y+z=7$

37) 10 students were selected from a school on the basis of values for giving awards and were divided into three groups. The first group comprises hard workers, the second group has honest and law abiding students and third group contains vigilant and obedient students. Double the number of students of the first group added to the number in the second group gives 13,while the combined strength of first and second group is four times that of the third group . Using matrix method ,find the number of students in each group . Apart from the values , hard work, honesty and respect for law, vigilance and obedience , suggest one more value , which in your opinion ,the school should consider for awards .

38) A school wants to award its students for the values of Honesty, Regularity and Hard work with a total cash award of ₹ 6,000. Three times the award money for Hard work added to that given for Honesty amounts to ₹ 11,000. The award money given for Honesty and Hard work together is double

the one given for Regularity. Represent the above situation algebraically and find the award money for each value, using matrix method. Apart from these values, namely, Honesty, Regularity and Hard work, suggest one more value which the school must include for awards.

39) Two cricket teams honoured their players for three values , excellent batting , to the point bowling and unparalleled fielding by giving ₹ x, ₹ y and ₹ z per player respectively. The first team paid respectively 2,2 and 1 players for the above values with a total prize money of ₹ 11 lakhs , while the second team paid respectively 1, 2 and 2 palyers for these values with a total prize money of ₹ 9 lakhs . If the total award money for one person each for each for these values amounts to ₹ 6 lakhs , then express the above situation as a matrix equation and find the award per person for each value. For which of the above mentioned values, would you like to pay more?

40) If $A=\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}$, find A^{-1} and use it to solve the system of equations:

$x+2y+z=4, -x+y+z=0, x-3y+z=2.$

41) Evaluate $\begin{vmatrix}\sqrt{13}+\sqrt{3} & 2\sqrt{5} & \sqrt{5}\\ \sqrt{15}+\sqrt{26} & 5 & \sqrt{10}\\ 3+\sqrt{65} & \sqrt{15} & 5\end{vmatrix}$.

42) Without expanding , evaluate $\begin{vmatrix}1 & \log_x y & \log_x z\\ \log_y z & 1 & \log_y z\\ \log_z x & \log_z y & 1\end{vmatrix}$; x, y, z being positive.

43) For a fixed positive Integer n, if $D=\begin{vmatrix}\underline{|n} & \underline{|n+1} & \underline{|n+2}\\ \underline{|n+1} & \underline{|n+2} & \underline{|n+3}\\ \underline{|n+2} & \underline{|n+3} & \underline{|n+4}\end{vmatrix}$, then $\left(\frac{D}{(\underline{|n})^3}-4\right)$ is divisible by n.

44) If a ≠ p, b ≠ q , r ≠ c and $\begin{vmatrix}p & b & c\\ a & q & c\\ a & b & r\end{vmatrix}=0$,find the value of $\frac{p}{p-a}+\frac{q}{q-b}+\frac{r}{r-c}$.

45) Let the three digit numbers A28, 3B9 and 62C, where A, B and C are integers between 0 and be divisible by a fixed integer k. Show that the determinant $\begin{vmatrix}A & 3 & 6\\ 8 & 9 & C\\ 2 & B & 2\end{vmatrix}$ is divisible by k.

46) Find the values of x satisfying $\begin{vmatrix}1 & 1 & \sin 3x\\ -4 & 3 & \cos 2x\\ 7 & -7 & -2\end{vmatrix}=0.$

47) If $A=\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & p & 1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & q \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$, then find the values of p and q.

48) If a , b, c (all positive) are the t p th , q th and r th terms respectively of G.P. ,then prove that

$$\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0.$$

49) Prove that $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix} = -2.$

50) If $f(x) = \dfrac{\sqrt{2}\cos x - 1}{\cot x - 1}, x \neq \dfrac{\pi}{4}$, find the value of $f\left(\dfrac{\pi}{4}\right)$ so that function f becomes continuous at $x = \dfrac{\pi}{4}$.

51) For what value of k is the function.

$$f(x) = \begin{cases} (x-1)\tan\dfrac{\pi}{2}x, & x \neq 1 \\ k, & x=1 \end{cases}$$ continuous at x=1 ?

52) If the function f defined by f (x) = $\begin{cases} \dfrac{1-\cos 4x}{x^2} & , x < 0 \\ a & , x=0 \\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4} & , x > 0 \end{cases}$ is continuous at x =0 find a

53) Find the values of ‘a’ for which the function f defend as $f(x) = \begin{cases} a\sin\dfrac{\pi}{2}(x+1) & ,x \leq 0 \\ \dfrac{\tan x - \sin x}{x^3} & ,x > 0 \end{cases}$ is continuous at x=0.

54) If $f(x)=\begin{cases} \dfrac{x-4}{|x-4|}+a & ,if\ \ x<4 \\ a+b & ,\text{if } x=4 \\ \dfrac{x-4}{|x-4|}+b & ,\text{if } x>4 \end{cases}$ is continuous at x=4, then find values of a and b.

55) If the function f defined by If a $=\begin{cases} \dfrac{1-\sin^3 x}{3\cos^2 x} & ,x<\dfrac{\pi}{2} \\ a & ,x=\dfrac{\pi}{2} \\ \dfrac{b(1-\sin x)}{(\pi-2x)^2} & ,x>\dfrac{\pi}{2} \end{cases}$ is continue at x==$\dfrac{\pi}{2}$ find a & b.